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Calcul d'integrale:

Exercice 1: Calculer les integrales suivants:

$A = \int_0^{\pi} \cos^2 x \, dx$ $\qquad B = \int_0^{\frac{\pi}{2}} x \cos x \, dx$

$C = \int_0^{\frac{\pi}{2}} (2x+3) \sin x \, dx$ $\qquad D = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x - \sin x}{x^2} \, dx$

$$E = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^3 x \, dx$$

$A = \int_0^{\pi} \cos^2 x \, dx$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\Rightarrow A = \int_0^{\pi} \frac{1 + \cos 2x}{2} \, dx$

$A = \left[\frac{x + \frac{1}{2} \sin 2x}{2} \right]_0^{\pi}$

$= \left[\frac{\pi + \frac{1}{2} \sin 2\pi}{2} - \frac{0 + \frac{1}{2} \sin 2 \times 0}{2} \right]_0^{\pi}$

$A = \frac{\pi}{2}$

$B = \int_0^{\frac{\pi}{2}} x\cos x \, dx$

On utilise une I.P.P : on pose : $u(x) = x$ $\quad \Leftrightarrow \quad$ $u'(x) = 1$

$v'(x) = \cos x \quad\quad v(x) = \sin x$

$\int uv' = [uv] - \int u'v$

$\Rightarrow B = [x\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx$

$= [x\sin x]_0^{\frac{\pi}{2}} - [-\cos x]_0^{\frac{\pi}{2}}$

$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} - (0 + \cos 0)$

$B = \frac{\pi}{2} - 1$

$C = \int_0^{\frac{\pi}{2}} (2x+3)\sin x \, dx$

On utilise une I.P.P : on pose : $u(x) = 2x+3$ $\quad \Leftrightarrow \quad$ $u'(x) = 2$

$v'(x) = \sin x \quad\quad v(x) = -\cos x$

$\int uv' = [uv] - \int u'v$

$\Rightarrow C = [-\cos x(2x+3)]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -2\cos x$

$= [-\cos x(2x+3)]_0^{\frac{\pi}{2}} - [-2\sin x]_0^{\frac{\pi}{2}}$

$= [-\cos x(2x+3) + 2\sin x]_0^{\frac{\pi}{2}}$

$= \left(-\cos\frac{\pi}{2}(\pi+3) + 2\sin\frac{\pi}{2}\right) - \left(-\cos 0(2\times 0+3) + 2\sin 0\right)$

$= 2+3$

$C = 5$

$$D = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x - \sin x}{x^2}\, dx$$

On peut remarquer que la fonction $x \mapsto \frac{\sin x}{x}$ est une primitive de

$$x \mapsto \frac{x\cos x - \sin x}{x^2}$$

$$\Rightarrow D = \left[\frac{\sin x}{x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

On peut aussi utiliser une I.P.P. On pose : $u(x) = x\cos x - \sin x \qquad u'(x) = -x\sin x$

$$v'(x) = \frac{1}{x^2} \Leftrightarrow v(x) = -\frac{1}{x}$$

$$\int uv' = [uv] - \int u'v$$

$$\Rightarrow D = \left[\frac{-x\cos x - \sin x}{x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{-x\sin x}{-x}\, dx$$

$$= \left[\frac{-x\cos x - \sin x}{x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \left[-\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{-x\cos x + \sin x + x\cos x}{x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\Rightarrow D = \left[\frac{\sin x}{x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$E = \int_{\pi/6}^{\pi/3} \sin^3 x \, dx = \int_{\pi/6}^{\pi/3} \sin x \left(1 - \cos^2 x\right) dx$$

$$= \int_{\pi/6}^{\pi/3} \sin x - \sin x \cos^2 x \, dx$$

$$= \left[-\cos x + \frac{1}{3} \cos^3 x \right]_{\pi/6}^{\pi/3}$$

$$= -\frac{1}{2} + \frac{1}{3} \times \frac{1}{8} + \frac{\sqrt{3}}{2} - \frac{1}{3} \times \left(\frac{\sqrt{3}}{2}\right)^3$$

$$E = \frac{-11 + 9\sqrt{3}}{24}$$